Reinforcement Learning for Supply Chains

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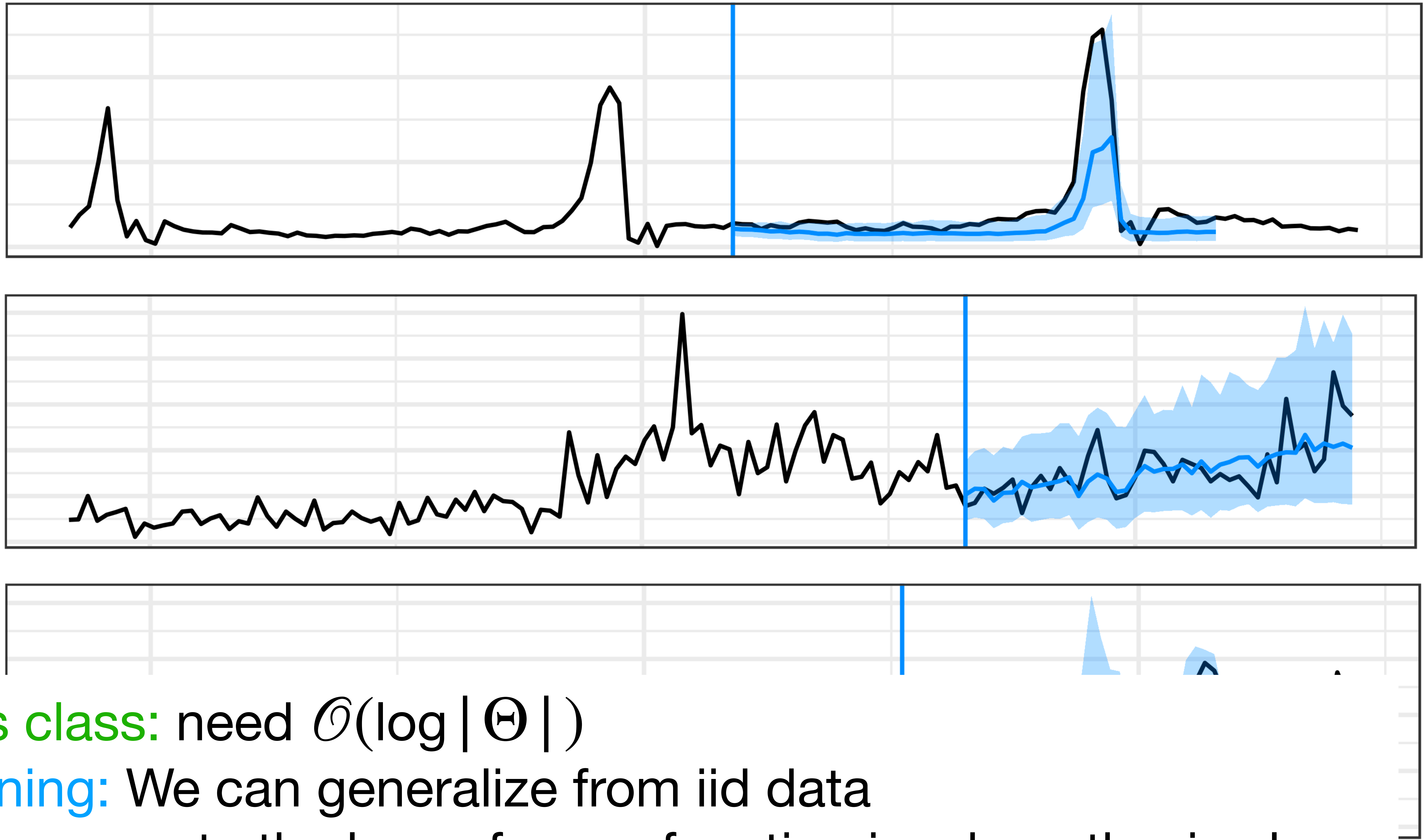
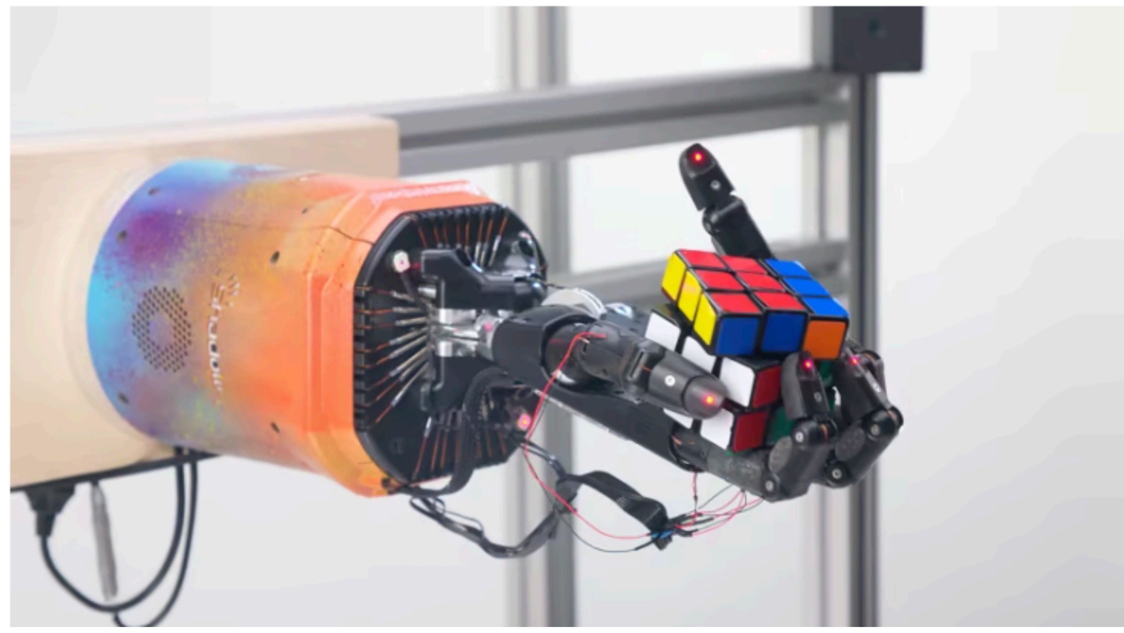
First, let’s look at supervised learning at Amazon.

See Wen, Torkkola, Narayanaswamy, M. (2017)
Eisenach, Patel, M. (2020)



- Finite hypothesis class: need $\mathcal{O}(\log|\Theta|)$
- Supervised Learning: We can generalize from iid data

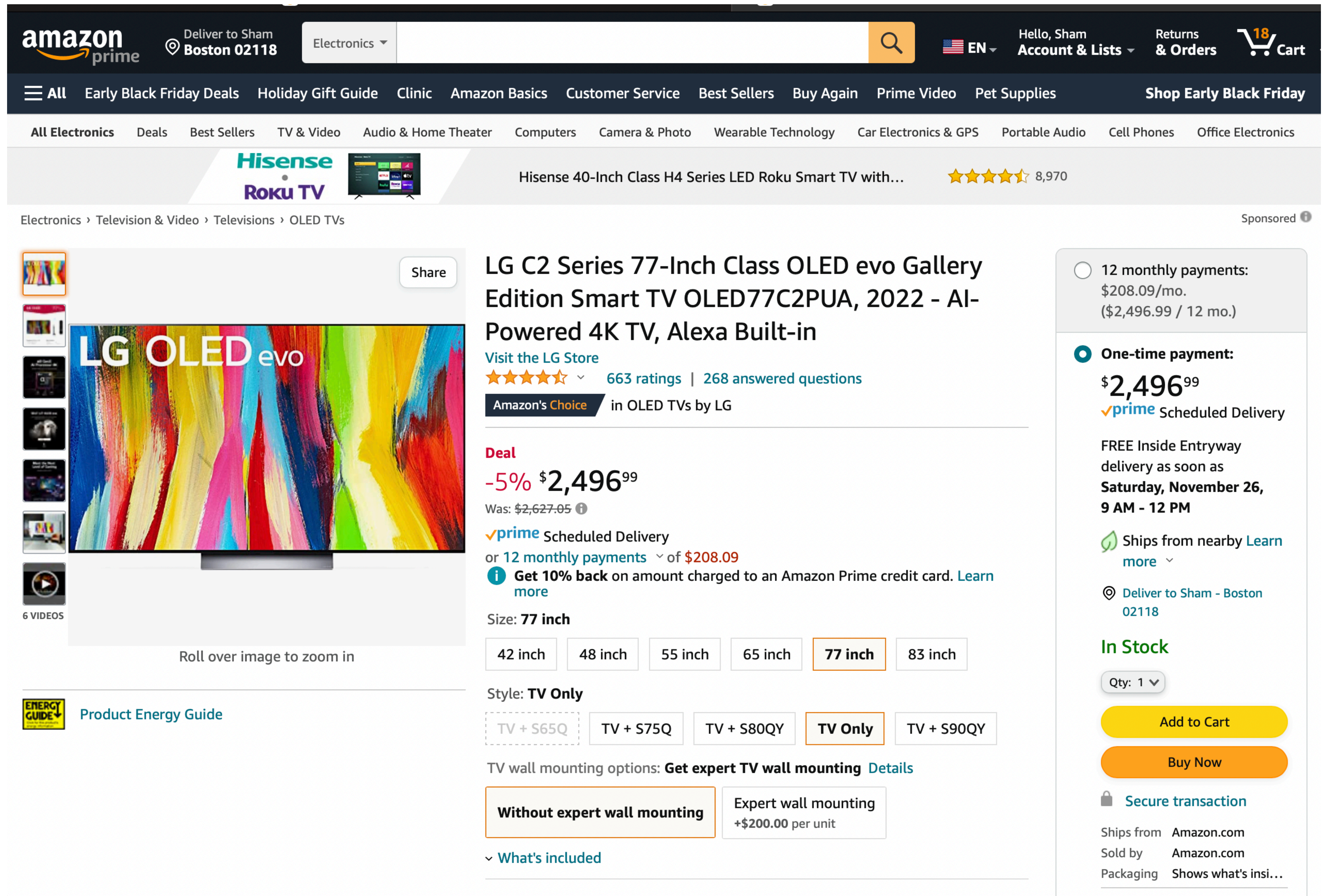
Data reuse: We can compute the loss of every function in a hypothesis class



Real-world RL is hard.

The core challenges Amazon faces are sequential decision making problems.

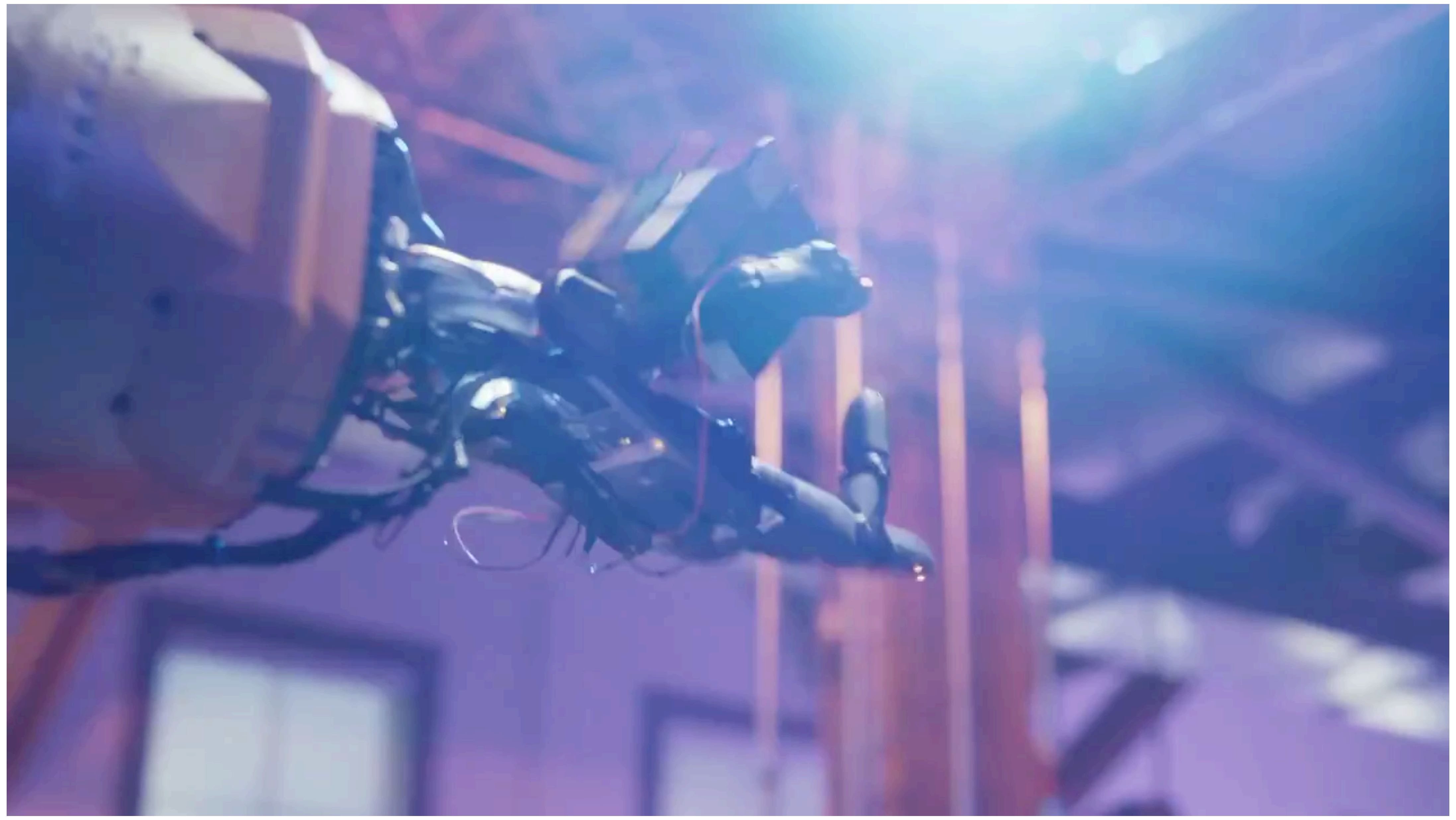
Can RL help in this space?



RL is hard!

- Sample complexity can be as large as min$(|\Theta|, 2^T)$
- Large state/action spaces
- Exploration
- Credit assignment problem

Dexterous Robotic Hand Manipulation
OpenAI, '19



The Supply Chain Problem

- Supply Chain is about buying, storing, and transporting goods.
- Amazon has been running it's Supply Chain for decades now
  - There is a lot of historical "off-policy" data
  - How do we use it?
  - Concern: counterfactual issue?
- This talk: how can we use this data to solve the inventory management problem?


The New York Times

Supply Chain Hurdles Will Outlast Pandemic, White House Says

The administration's economic advisers see climate change and other factors complicating global trade patterns for years to come.


The New York Times

How the Supply Chain Crisis Unfolded

Outline

Can we use historical data to solve inventory management problems in supply chain?

- Part I: Utilizing Historical Data
- Part II: Moving to real-world inventory management problems
- Part III: Real World Results

Deep Inventory Management

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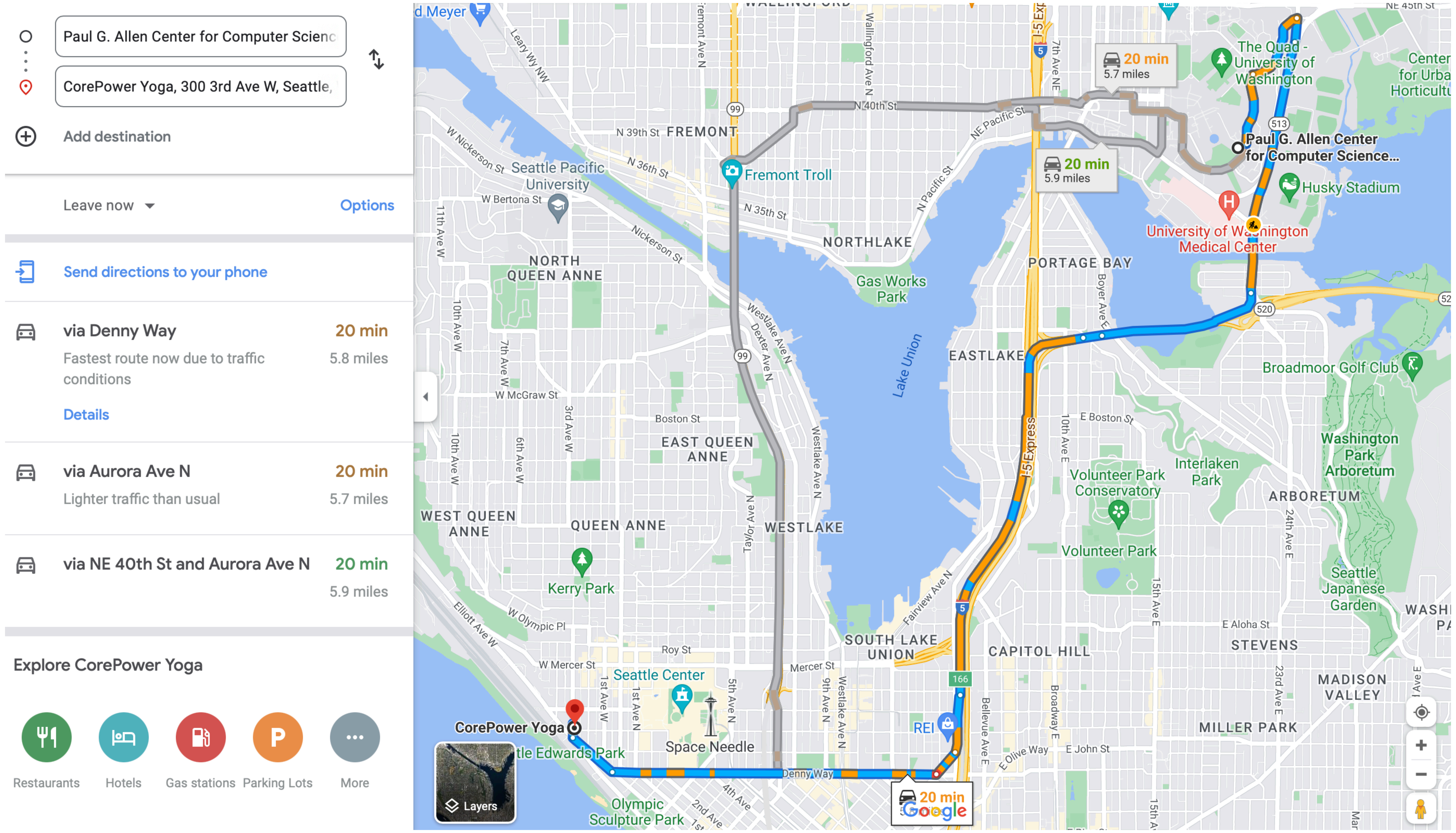
Largely based on this paper: arxiv/2210.03137

I: Utilizing historical data

Warm up: Vehicle Routing
(when using historical data might be ok)

- We want a good policy for routing a single car.

- Policy π: features -> directions
  features:
  time of day, holiday indicators, current traffic, sports games, accidents, location, weather,



- Historical Data:
  suppose we have logged historical data of features

- Backtesting policies:
  - Key idea: a single route minimally affects traffic
  - Counterfactual: with the historical data, we can see what would have happened with another policy.

Warm up 2: Fleet Routing

- We want to route a whole fleet of self-driving taxis.
- Policy π: features -> directions
  - features:
    customer demand, time of day, holiday indicators, current traffic, sports games, accidents, location, weather…
- Historical Data:
  suppose we have logged historical data of features
- Backtesting policies:
  - Key idea: a small fleet route may have small affects on traffic.
  - Counterfactual: with the historical data, we can see what would have happened with another policy.

Supply Chain Data

Price= $2
Cost= $1

Time	Inventory	Demand	Order	Revenue
0	100	20	-	40
0	80	-	10	-10
1	90	20	-	40
1	70	-	50	-50
2	120	60	-	120
2	60	-	10	-10

Backtesting a policy

Time	Inventory	Demand	Order	Revenue
0	100	20	-	40
0	80	-	~~10~~ 40	~~-10~~ -40
1	~~90~~ 120	20	-	40
1	~~70~~ 100	-	~~50~~ 20	~~-50~~ -20
2	120	60	-	120
2	60	-	10	-10

Price= $2
Cost= $1

- Current order doesn't impact future demand.
  - This allows us to backtest!
  - Empirically, backlog due to unmet demand does not look significant.[1]

1. See Verhoef et al (2006)

Formalization of the Supply Chain Problem

- Growing literature around a class of MDPs where a large part of the state is driven by an exogenous noise process [Efroni et al 2021, Sinclair et al 2022]

- A formalization of the model:
  - Action a_t: how much you buy
  - Exogenous random variables: evolving under $\Pr$ and not dependent on our actions $(\text{Demand}_t, \text{Price}_t, \text{Cost}_t, \text{Lead Time}_t, \text{Covariates}_t) := s_t$
  - Controllable part (inventory) I_t: evolution is dependent on our action.
    - $I_t = \max(I_{t-1} + a_{t-1} - D_t, 0)$ (and suppose we start at I_0).
  - Reward is just the sum of profits: $r(s_t, I_t, a_t) := \text{Price}_t \times \min(\text{Demand}_t, I_t) - \text{Cost}_t \times a_t$

- Learning setting:
  - Data collection: We observe N historical trajectories, where each sequence is sampled $s_1, \ldots, s_T \sim \Pr$
  - Goal: maximize our rewards cumulative reward over T periods

$$V_T(\pi) = E_\pi\Big[\sum_{t=1}^{T} \gamma^t r(s_t, I_t, a_t)\Big]$$

Why is it an interesting RL problem?

- Lots of time dependence!
  - If you buy too much, you're left with the inventory for months!
  - Your actions (orders) affect the state at a random time later
  - Tons of correlation across time (Demand, Price, Cost)
- We can explore (linear risk in every product)

Theorem: Backtesting in ExoMDPs

Theorem [M., Torkkola, Eisenach, Luo, Foster, Kakade '22]:
Suppose we have a set of K policies $\Pi = \{\pi_1, \ldots \pi_K\}$, and we have N sampled exogenous paths. Then we can accurately backtest up to nearly $K \approx 2^N$ policies.

Formally, for any $\delta \in (0,1)$, with probability greater than $1 - \delta$ - we have that for all $\pi \in \Pi$:

$$| V_T(\pi) - \hat{V}_T(\pi) | \leq T \sqrt{\frac{\log(K/\delta)}{N}}$$

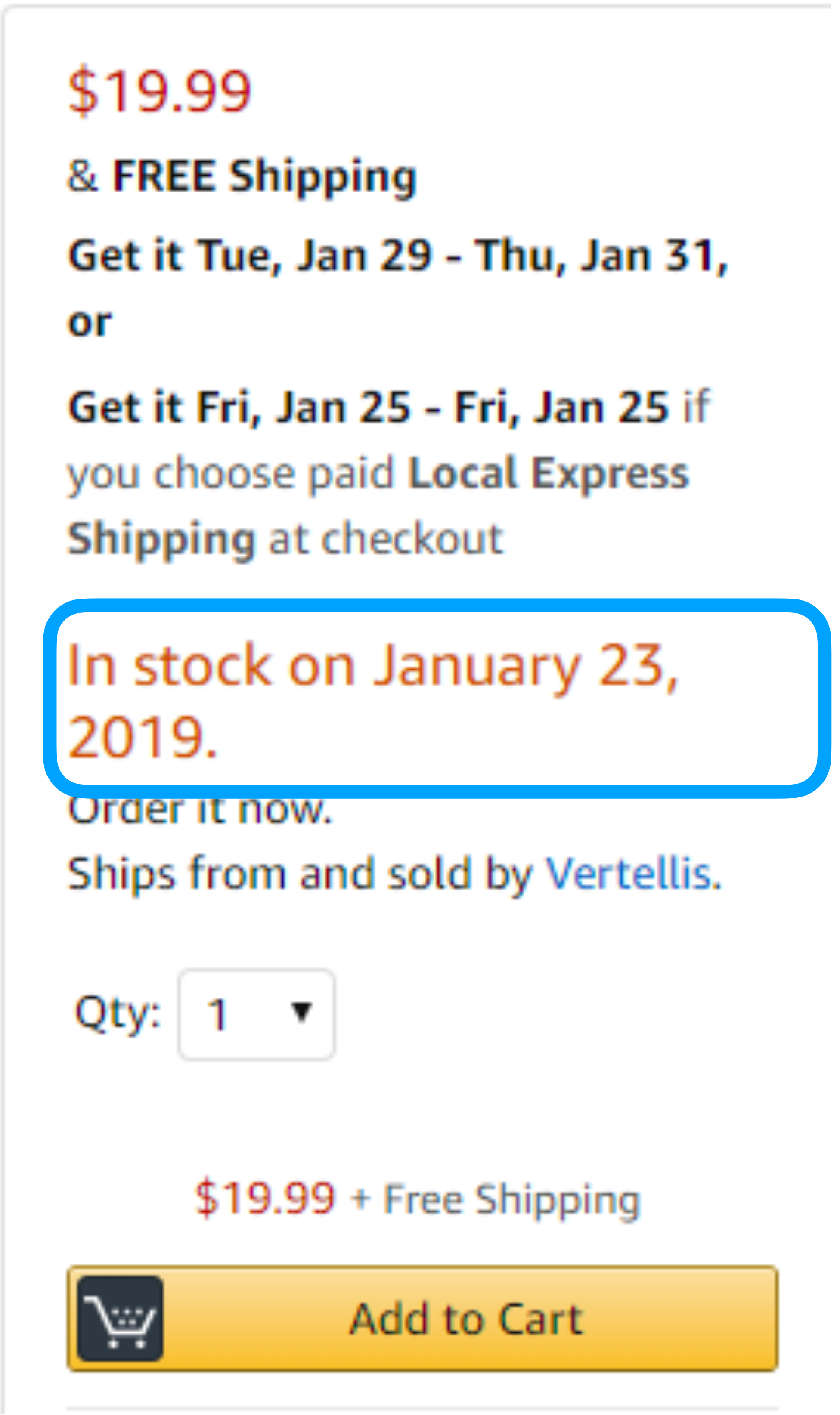
(assuming the reward r_t is bounded by 1).

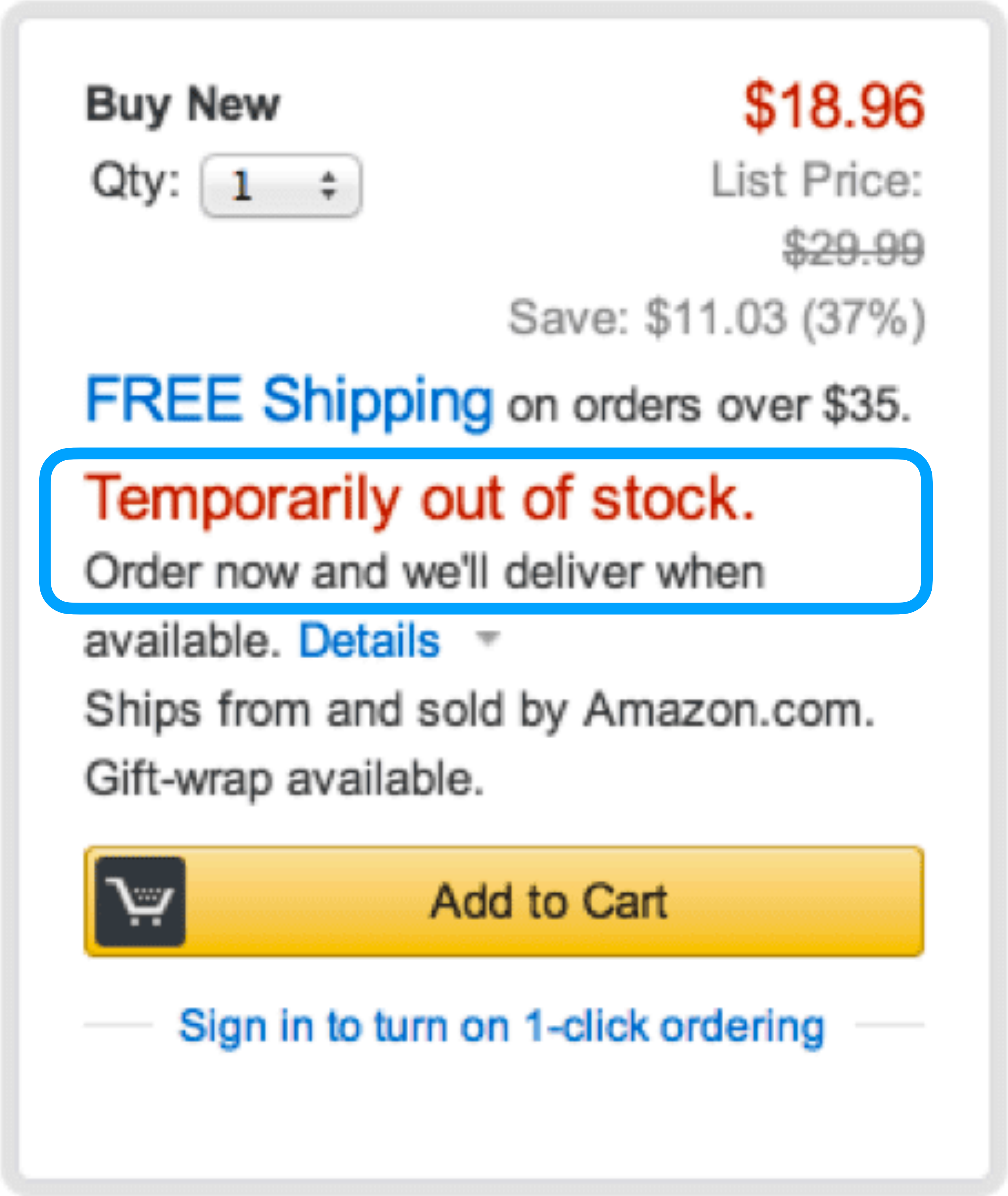
- Implications:
  - We can optimize a neural policy on the past data.
  - In the usual RL setting (not exogenous), we would have an amplification factor of (at least) $\min\{2^T, K\}$, using historical data due to the counterfactual issue.

II: Real World Inventory Management Problems

Real-world Issue: Censored Demand

- When demand $\geq$ inventory, what customers see:





We only observe **sales** not the **demand:**

Sales $:=$ min(Demand, Inventory)

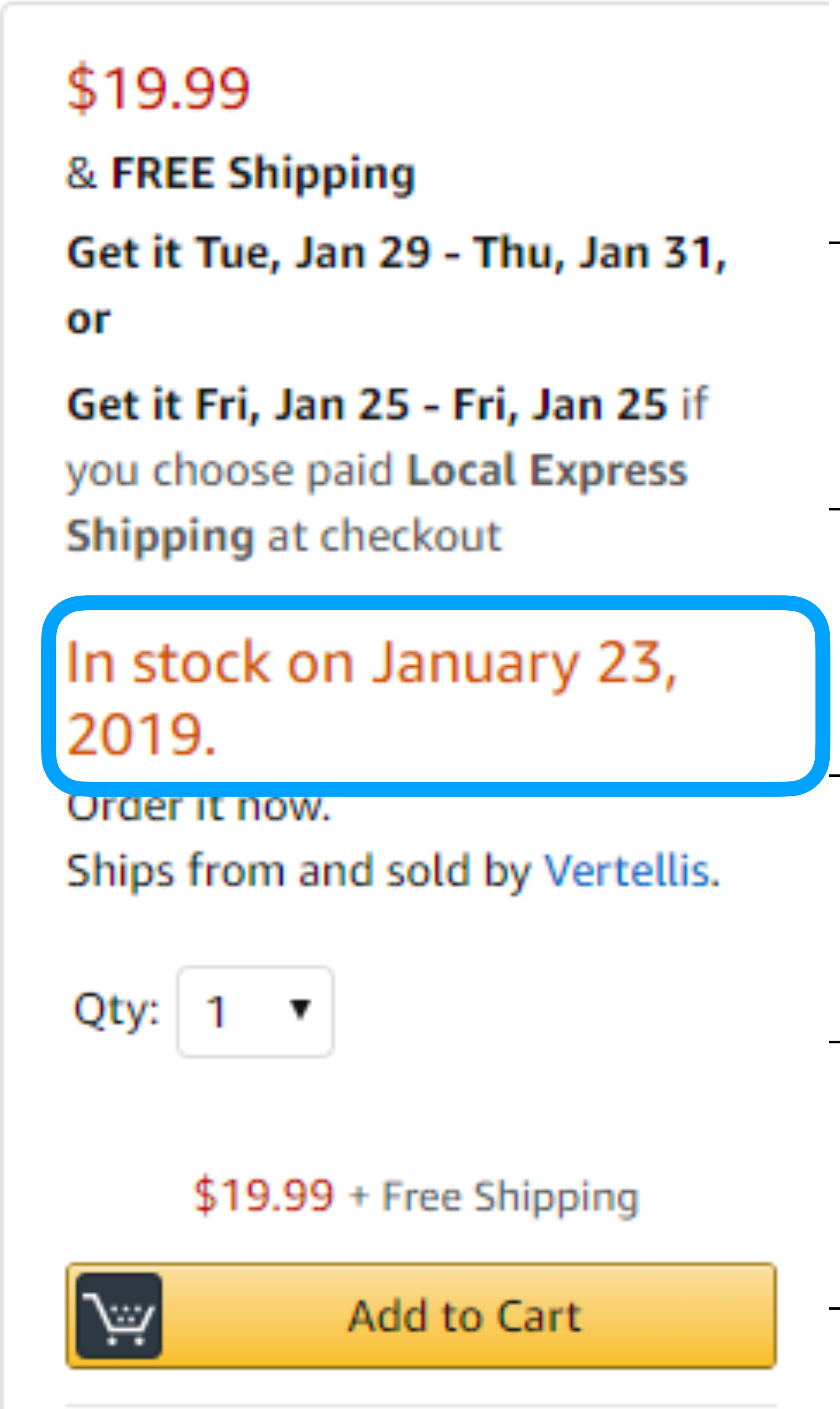
Can we still backtest?

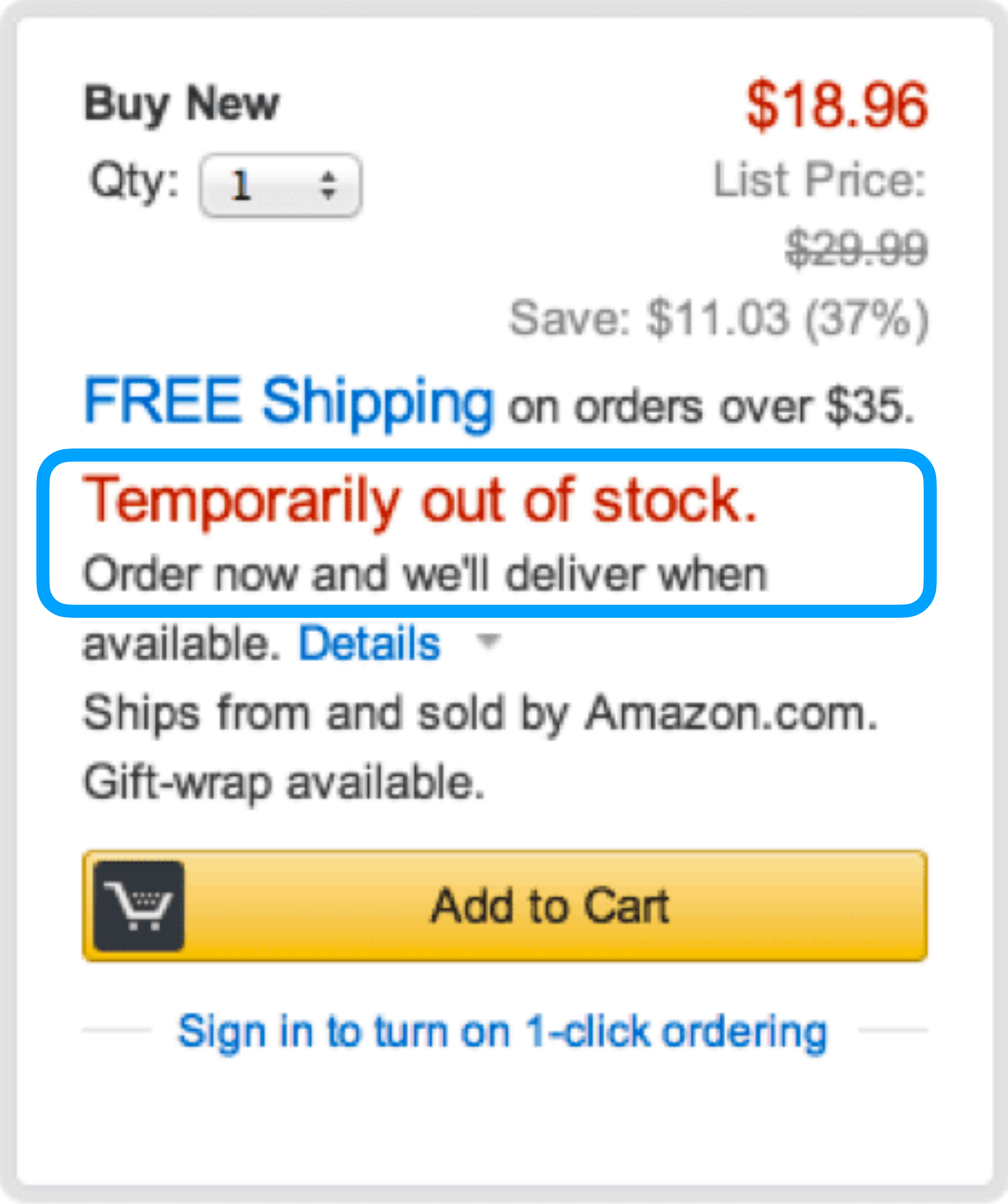
Our historical data is then censored….

$$\text{Sales} := \min(\text{Demand}, \text{Inventory})$$

Price= $2
Cost= $1

Time	Inventory	True Demand	Sales	Order	Revenue
T	10	??	10	-	20
⋮	⋮				
⋮	⋮				
⋮	⋮				
⋮	⋮				
⋮	⋮				



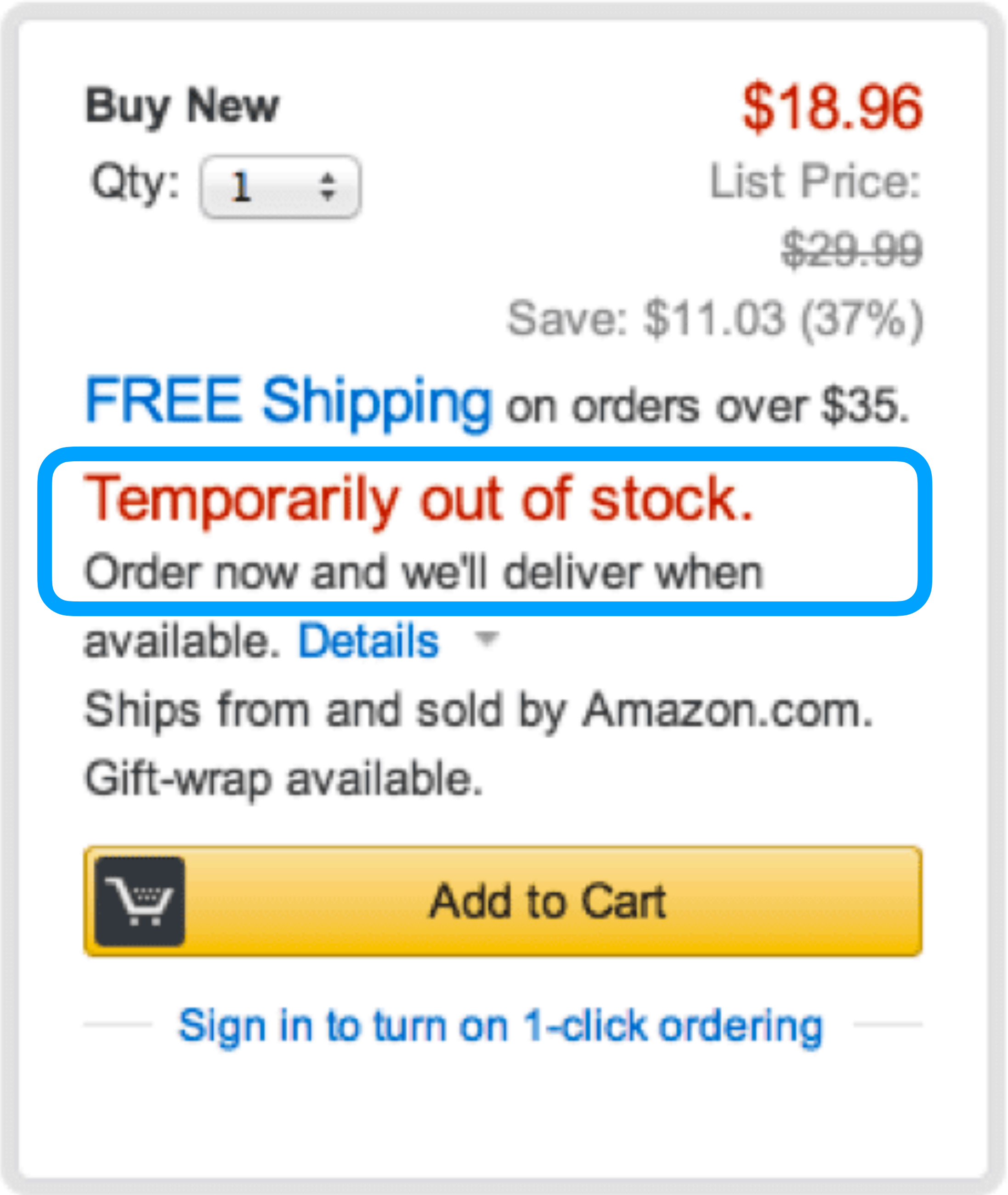


If we could fill in the missing demand, then we could still backtest!

We have many observed historical covariates

- Covariates:
  Sales, Web Site, Glance Views, Product Text, Reviews

- Example: the #times customers look at an item gives us info about the unobserved demand.



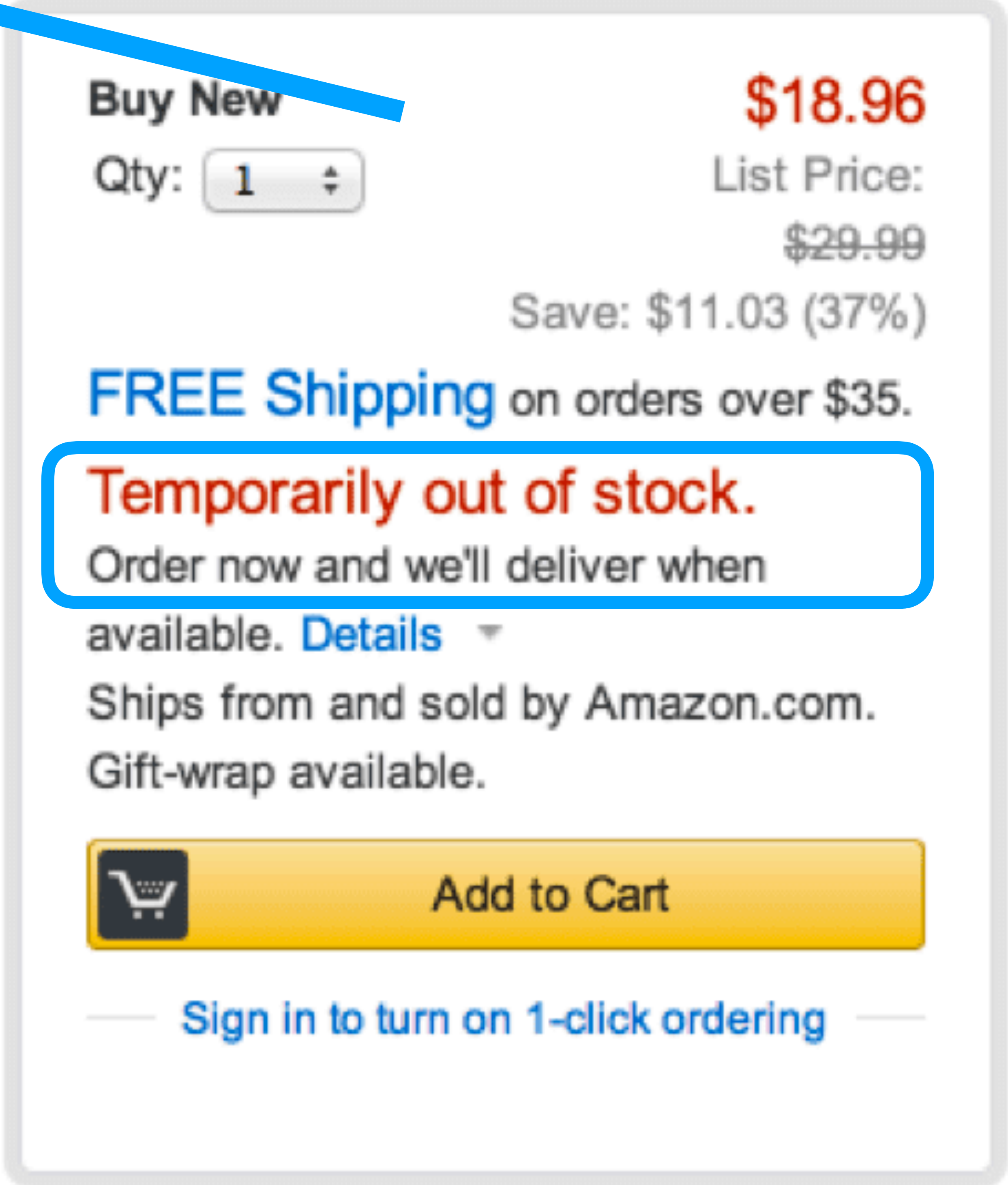
- Let's forecast the missing variables from the observed covariates!
  $\hat{\mathbb{P}}(\text{Missing Data} \mid \text{Observed Data})$

Uncensoring the data….

Sales := min(Demand, Inventory)

Price= $2
Cost= $1

Time	Inventory	True Demand	Sales	Order	Revenue
T	10	**40**	10	-	20
⋮	⋮	⋮	⋮		
⋮	⋮	⋮	⋮		
⋮	⋮	⋮	⋮		
⋮	⋮	⋮	⋮		
⋮	⋮	⋮	⋮		



Key idea:
Use covariates (e.g. glance views) to forecast missing demand, vendor lead times, etc

Theorem: Backtesting in Uncensored ExoMDPs

Theorem [M., Torkkola, Eisenach, Luo, Foster, Kakade 22]:
If we can forecast the missing variables accurately (in a total variation sense), then we can backtest accurately. More formally,

Setting: we have N sampled sequences $\{s_1^i, s_2^i, \ldots s_T^i\}_{i=1}^N$,
where M_i and O_i are the missing and observed exogenous variables in sequence i.

Forecast: $\widehat{\mathbb{P}}^i = \widehat{\Pr}(M_i \mid O_i)$ is our forecast of $\mathbb{P}^i = \Pr(M_i \mid O_i)$.

Assume: With pr. 1, forecasting has low error: $\frac{1}{N} \sum_{i=1}^{N} \text{TotalVar}\left(\mathbb{P}^i, \widehat{\mathbb{P}}^i\right) \leq \epsilon_{\text{sup}}$.

Guarantee: For any $\delta \in (0,1)$, with pr. greater than $1 - \delta$, for all $\pi \in \Pi$:

$$|V_T(\pi) - \hat{V}_T(\pi)| \leq T\left(\epsilon_{\text{sup}} + \sqrt{\frac{\log(K/\delta)}{N}}\right)$$

- Key idea: We can backtest even in the censored setting!

III: Training Policies & Empirical Results

The Simulator

- Collection of historical trajectories:
  - 1 million products
  - 104 weeks of data per product

Data

- Uncensoring:
  - Demand
  - Vendor Lead Times

Corrections

- Policy gradient methods in a “gym”:
  - “gym” ↔ backtesting ↔ simulator
    (note the “simulator” isn’t a good world model).
  - The policy can depend on many features.
    (seasonality, holiday indicators, demand history, ASIN, text features)

Simulator

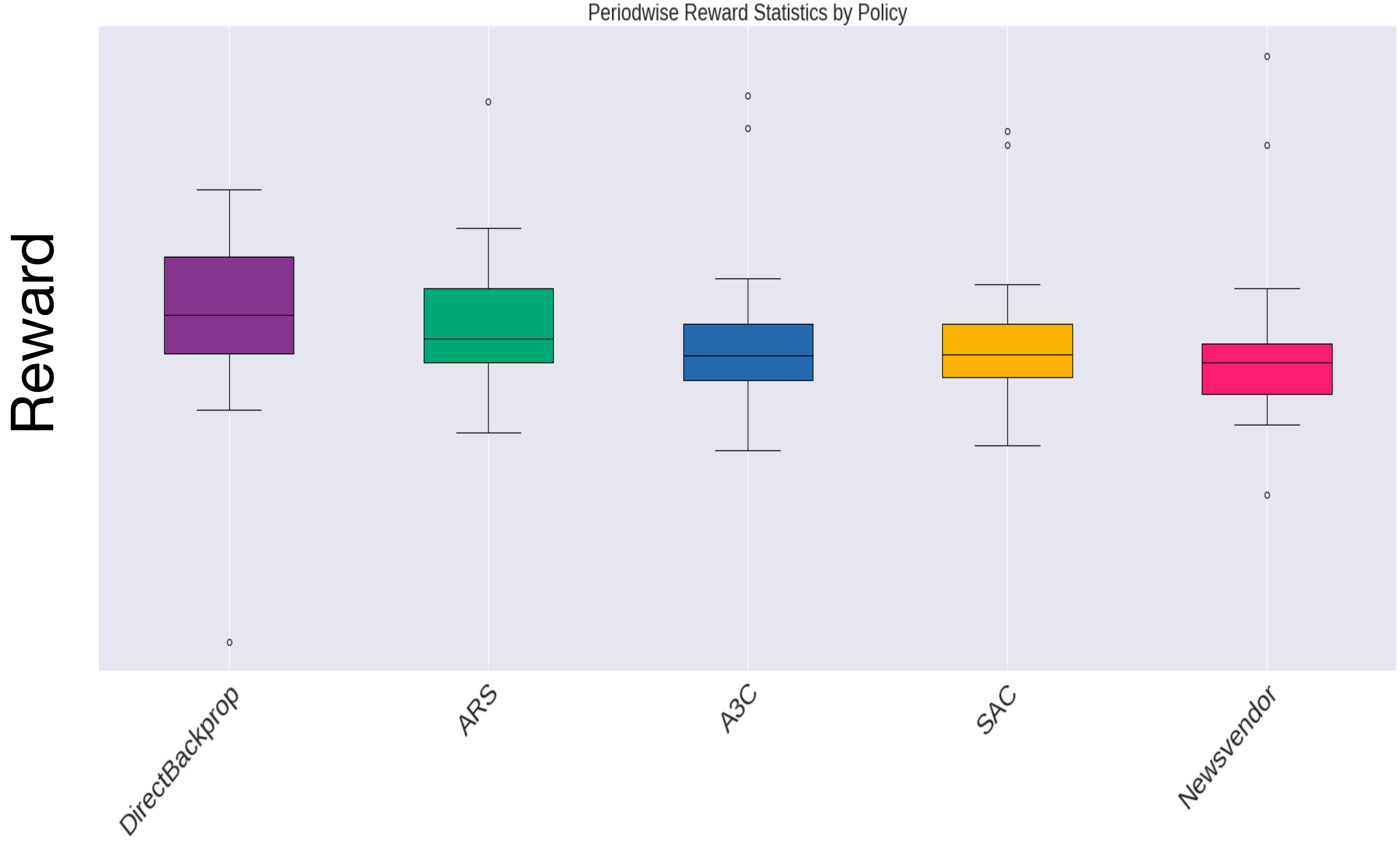
Differentiable Control Problem

- Note that each term of our state evolution is a differentiable function of previous actions
- So, we can take gradients directly from our Reward through our policy
- This is our current production policy, called *DirectBackprop*
- Similar in spirit to Perturbation Analysis (Glasserman et al 1995), except it uses a neural policy

Sim to Real Transfer

- Sim: the backtest of DirectBackprop improves on Newsvendor.
- Real: DirectBackprop significantly reduces inventory without significantly reducing total revenue.

Simulation



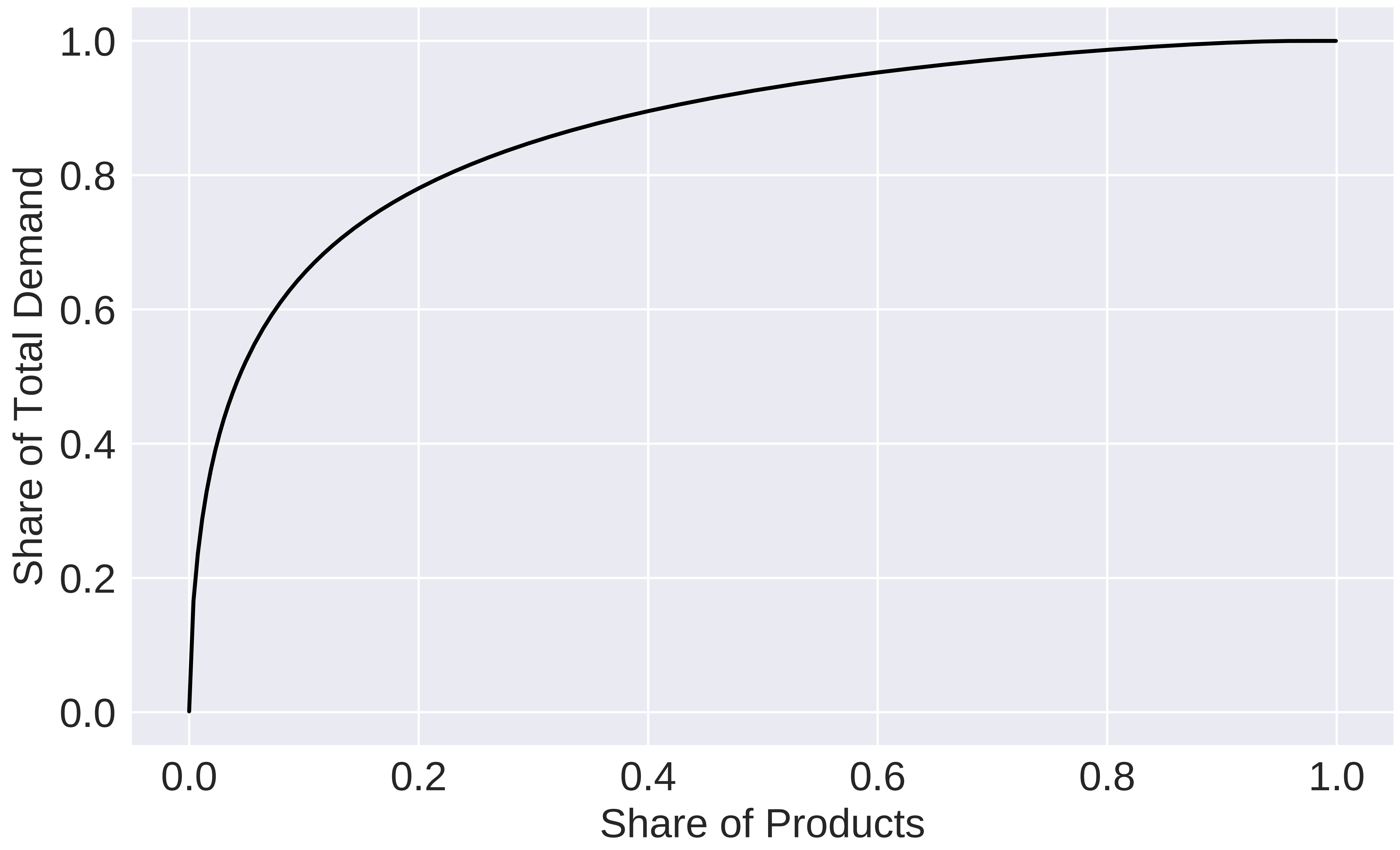
Real World

Metrics	% change
Inventory Level	**-12** $\pm$ 6
Revenue	$\sim$

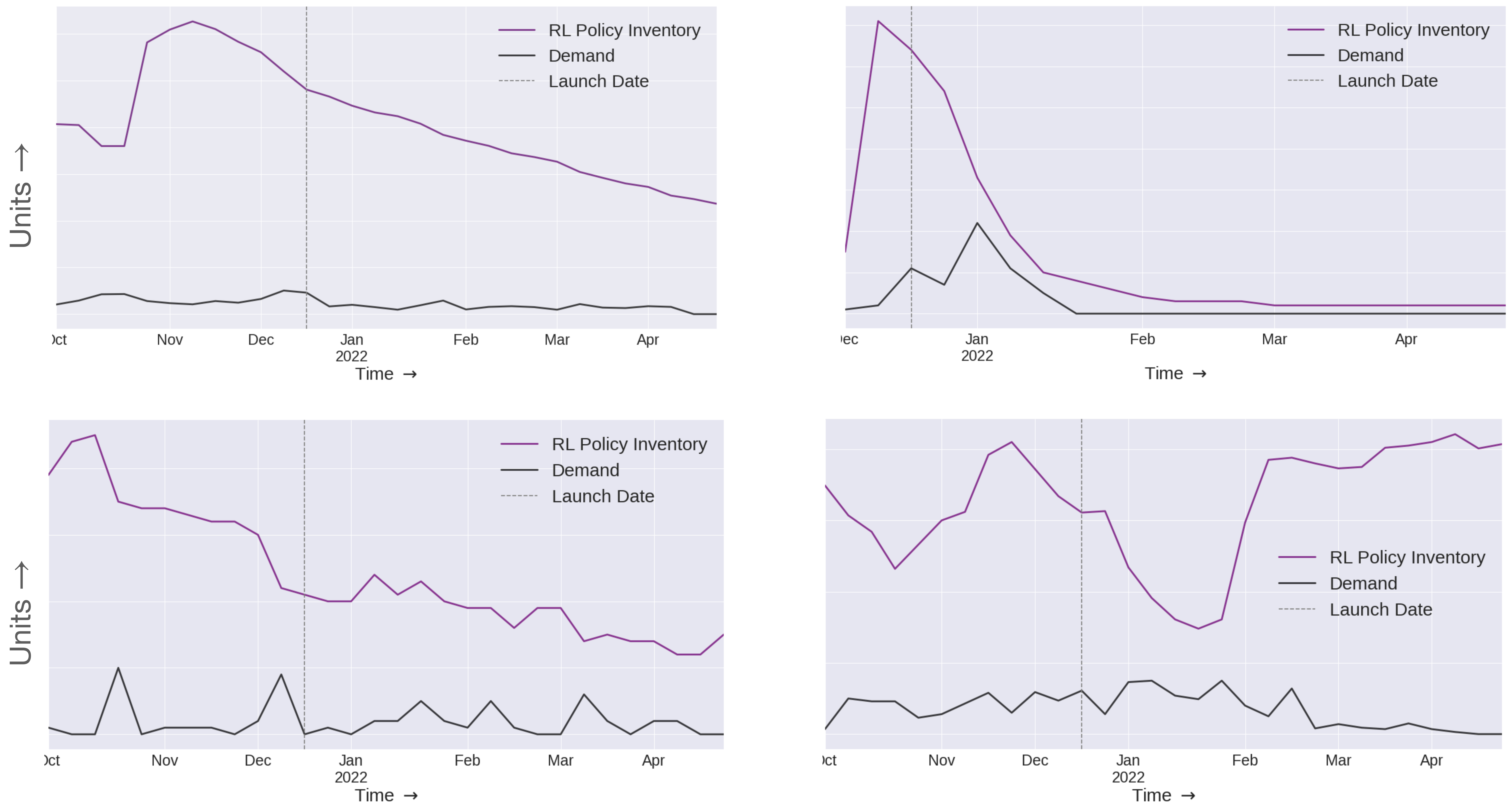
What about in the real world?

- Really hard to measure! (Tripuraneni, M et. al 2021)

- Heavy tailed data:
    - A few products contribute to most of the reward



Anecdotally, RL has reasonable strategies in the real world…



Real World RL Challenges

- World is not perfectly exogenous (some terms may depend on our actions)
- Cross product constraints are computationally intensive
- Not every Supply Chain problem can be written in this framework

Conclusion

- There are a class of RL Problems that work in the real world!

- The exogenous assumption allows us to backtest **any** policy on historical data

- A large number of classical Operations Research problems fall into this class of Interactive Decision-Making problems

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Conclusion

- There are a class of RL Problems that work in the real world!
- The exogenous assumption allows us to backtest **any** policy on historical data
- A large number of classical Operations Research problems fall into this class of Interactive Decision-Making problems





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