Deterministic Calibration with Simpler Checking Rules

Dean P. Foster / Sham Kakade
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	Individual calibration	**Public calibration**
Predicting actions X_i = other people's actions p_i = forecast of X_i	$\to$ ϵ-CE in $(1/\epsilon)^{a^n}$ rounds $\frac{1}{T}\sum_{t=1}^{T}(X_t - p_t)w(p_t) \to 0$	$\to$ ϵ-NE. Naive in $(1/\epsilon)^{na^n}$ rounds, improved in $(1/\epsilon)^{a^n}$. $\frac{1}{T}\sum_{t=1}^{T}(X_t - p_t)w(\vec{p_t}) \to 0$
Predicting utililities X_i = my utility p_i = forecast of X_i	$\to$ ϵ-CE in $(1/\epsilon)^{a}$ rounds $\frac{1}{T}\sum_{t=1}^{T}(X_t - p_t)w(p_t) \to 0$	$\to$ ϵ-NE in $(1/\epsilon)^{an}$ rounds $\frac{1}{T}\sum_{t=1}^{T}(X_t - p_t)w(\vec{p_t}) \to 0$
Neighbors' actions X_i = actions of neighbors p_i = forecast of X_i	no-speedup over public	$\to$ ϵ-NE in $(1/\epsilon)^{da^d}$ rounds $\frac{1}{T}\sum_{t=1}^{T}(X_t - p_t)w(\vec{p}_{N(i)-i,t}) \to 0$
Graphs/utilities X_i = my utility p_i = forecast of X_i	no-speedup over public	$\to$ ϵ-NE in $(1/\epsilon)^{da}$ rounds $\frac{1}{T}\sum_{t=1}^{T}(X_t - p_t)w(\vec{p}_{N(i)-i,t}) \to 0$
Linear regrets X_i = my utility p_i = forecast of X_i	$\to$ CE in $(1/\epsilon)^2$ rounds $\frac{1}{T}\sum_{t=1}^{T}(X_t - p_t)l(p_t) \to 0$	conjecture: $\to$ NE? In $(1/\epsilon)^2 a^n$? $\frac{1}{T}\sum_{t=1}^{T}(X_t - p_t)l(\vec{p}, \vec{p}^2, \ldots, \vec{p}^n) \to 0$
Trusting neighbors X_i = my play, p_i = forecast of X_i. There is no protection against failure to follow the protocol.	Doesn't converge?	$\to$ ϵ-NE in $(1/\epsilon)^{da}$ rounds $\frac{1}{T}\sum_{t=1}^{T}(X_t - p_t)w(\vec{p}_{N(i)-i,t}) \to 0$

General information

At each round, players compute $p_{i,t}$ and use it to make their decision of what to do in round t. They always use a smooth best reply function. In all but the "trusting neighbors" rule, they will be guarenteed to have good forecasts (i.e. calibrated) to base their decisions on.

ϵ = target accuracy
n = number of players
a = number of actions each player has
d = number of neighbors each player has
$w()$ = smooth bump function
$l()$ = linear function

CALIBRATION

- **Origin of calibration**
  - Dawid asked whether calibration existed: Dawid, A. P. (1985) "The well calibrated Bayesian." *JASA*.
  - Oakes answered no: Oakes, D. (1985) "Self-calibrating priors do not exist," *JASA*.
- **Blackwell approachability**
  - Blackwell, David (1956) "An Analog of the Minimax Theorem for Vector Payoffs," *Pacific Journal of Mathematics,* **6**. (Easier to find is Luce and Raiffa *Games and Decisions* Appendix 8.6, p 479 - 483.)
  - For a review paper that discusses how to use Blackwell for calibration, see: Foster, D. and R. Vohra (1999) "Regret in the On-line decision problem," *Games and Economic Behavior*, 7-36.
- **No regret and calibration**
  - Foster and Vohra's first proof of the existence of calibration/no-regret (in 1991) wasn't convincing. Eventually it came out as: Foster, D. and R. Vohra (1998) "Asymptotic Calibration," with R. Vohra, *Biometrika*, 379 - 390.
  - Other proofs
    - Fudenberg, D. and D. Levine (1999) "An easier way to calibrate," *Games and Economic Behavior.*
    - Foster, D. (1999) "A proof of calibration via Blackwell's Approachability theorem," *GEB.*

LEARNING in GAMES

- **Calibration converges to CE**
  - "Calibrated Learning and Correlated Equilibrium," with R. Vohra *Games and Economic Behavior*, (1997) **21**, 40-55.
  - Hart, S. and A. Mas-colell (2000) "A simple adaptive procedure leading to correlated equilibrium," *Econometrica*, **68**, 1127 - 1150.
- **Exhastive search converges to NE**
  - "Learning, Hypothesis Testing and Nash Equilibrium," with H. P. Young, *Games and Economic Behavior*, 2003, 73 - 96.
  - "Regret Testing: A simple payoff-based procedure for learning Nash equilibrium," with H. Young, under revision for *JET*.
  - Germano, Fabrizio and Gabor Lugosi, 2005, "Global Nash convergence of Foster and Young's regret testing," mimeo.
- **Public calibration converges to NE**
  - "Deterministic Calibration and Nash Equilibrium" with with Sham M. Kakade, 2004, *COLT.*
  - This talk. Coming soon on our web pages:
    - http://gosset.wharton.upenn.edu/∼foster/
    - http://www.cis.upenn.edu/∼ skakade/

Contact information: dean@foster.net / skakade@linc.cis.upenn.edu.